Passivity-Preserving Balanced Truncation Model Reduction of Circuit Equations

Tatjana Stykel and Timo Reis

Abstract We consider passivity-preserving model reduction of circuit equations using the bounded real balanced truncation method applied to a Moebius-transformed system. This method is based on balancing the solutions of the projected Lur'e or Riccati matrix equations. We also discuss their numerical solution exploiting the underlying structure of circuit equations. A numerical example is given.

1 Introduction

A modified nodal analysis (MNA) for linear RLC circuits yields a linear system of differential-algebraic equations (DAEs)

$$\begin{aligned} E\dot{x}(t) &= Ax(t)+Bu(t),\\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where

$$E=\begin{bmatrix} A_{\mathcal{C}}\mathcal{C}A_{\mathcal{C}}^T & 0 & 0\\ 0 & \mathcal{L} & 0\\ 0 & 0 & 0 \end{bmatrix},\quad A=\begin{bmatrix} -A_{\mathcal{R}}\mathcal{R}^{-1}A_{\mathcal{R}}^T & -A_{\mathcal{L}} & -A_{\mathcal{V}}\\ A_{\mathcal{L}}^T & 0 & 0\\ A_{\mathcal{V}}^T & 0 & 0 \end{bmatrix},\quad B=-\begin{bmatrix} A_I & 0\\ 0 & 0\\ 0 & I \end{bmatrix}=C^T. \tag{2}$$

Here $A_{\mathcal{C}}\in\mathbb{R}^{n_\eta,n_{\mathcal{C}}}$, $A_{\mathcal{L}}\in\mathbb{R}^{n_\eta,n_{\mathcal{L}}}$, $A_{\mathcal{R}}\in\mathbb{R}^{n_\eta,n_{\mathcal{R}}}$, $A_{\mathcal{V}}\in\mathbb{R}^{n_\eta,n_{\mathcal{V}}}$ and $A_I\in\mathbb{R}^{n_\eta,n_I}$ are incidence matrices describing the circuit topology, and $\mathcal{R}$, $\mathcal{L}$ and $\mathcal{C}$ are resistance, inductance and capacitance matrices, respectively. Linear RLC circuits are often used to model interconnects, transmission lines and pin packages in VLSI networks.

In the following we will assume that

- the matrix $A_{\mathcal{V}}$ has full column rank;
- the matrix $[A_{\mathcal{C}},A_{\mathcal{L}},A_{\mathcal{R}},A_{\mathcal{V}}]$ has full row rank;
- the matrices $\mathcal{R}$, $\mathcal{L}$ and $\mathcal{C}$ are symmetric and positive definite.

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These assumptions guarantee that the pencil $\lambda E - A$ is regular, i.e., $\det(\lambda E - A) \not\equiv 0$. Moreover, system (1), (2) is *passive*, i.e., it does not generate energy, and *reciprocal*, i.e., its transfer function $G(s) = C(sE - A)^{-1}B$ satisfies the symmetry relation $G(s) = S_{\text{ext}} G(s)^T S_{\text{ext}}$ with an external signature $S_{\text{ext}} = \operatorname{diag}(I_{n_l}, -I_{n_\nu})$, see [1]. Furthermore, passivity is equivalent to the *positive realness* of G meaning that G is analytic in the open right half-plane $\mathbb{C}_+$ and $G(s) + G^T(\overline{s})$ is positive semidefinite for all $s \in \mathbb{C}_+$, see [2].

The number $n = n_\eta + n_\lambda + n_\nu$ of state variables in (1) is related to the number of circuit elements and usually very large. This makes the analysis and numerical simulation of circuit equations unacceptably time consuming. Therefore, model order reduction is of great importance.

A general idea of model reduction is to approximate the large-scale system (1) by a reduced-order model

$$\begin{aligned} \tilde{E}\,\dot{\tilde{x}}(t) &= \tilde{A}\tilde{x}(t) + \tilde{B}u(t), \\ \tilde{y}(t) &= \tilde{C}\tilde{x}(t), \end{aligned} \tag{3}$$

where $\tilde{E}$, $\tilde{A} \in \mathbb{R}^{\ell,\ell}$, $\tilde{B} \in \mathbb{R}^{\ell,m}$, $\tilde{C} \in \mathbb{R}^{m,\ell}$ and $\ell \ll n$. It is required that the approximate system (3) captures the input-output behaviour of (1) to a required accuracy and preserves passivity and reciprocity. The preservation of these properties allows a back interpretation of the reduced-order model (3) as an electrical circuit which has fewer electrical components than the original one [1, 2].

Krylov subspace based methods [3, 4] are mostly used model reduction methods in circuit simulation. Although these methods are efficient for very large sparse problems, stability and passivity are not necessarily preserved in the reduced-order model. Passivity-preserving model reduction methods based on Krylov subspaces have been developed for standard state space systems [5, 6] and also for structured generalized state space systems describing interconnect circuits [4, 7, 8]. Despite the successful application of these methods in circuit simulation, they provide only a good local approximation and, so far, there exist no global error bounds.

Balanced truncation is another model reduction approach commonly used in control design. In order to capture specific system properties, different balancing techniques have been developed for standard state space systems, e.g., [9, 10] and also for DAEs [11, 12]. An important property of balancing-related model reduction is the existence of computable error bounds. Balanced truncation is based on the transformation of the dynamical system into a balanced form whose controllability and observability Gramians are both equal to a diagonal matrix. Then a reduced-order model is determined by the truncation of the states corresponding to small diagonal elements of the balanced Gramians.

In this paper, we present a passivity-preserving model reduction method for circuit equations (1), (2) that is based on so-called bounded real balanced truncation applied to a Moebius-transformed system. It requires balancing two Gramians that satisfy the projected Lur'e equations. Under some assumptions such equations can be rewritten as the projected Riccati equations. We also discuss the numerical solution of these matrix equations via Newton's method and present some results of numerical experiments.

Throughout the paper $\mathbb{R}^{n,m}$ denotes the spaces of $n \times m$ real matrices and A^T stands for the transpose of $A \in \mathbb{R}^{n,m}$. An identity matrix of order n is denoted by I_n or simply by I. Further, for symmetric matrices $X, Y \in \mathbb{R}^{n,n}$, we write $X > Y$ ($X \geq Y$) if $X - Y$ is positive (semi)definite. For a real diagonal matrix $D = \mathrm{diag}(d_1, \ldots, d_n)$, we have $|D| = \mathrm{diag}(|d_1|, \ldots, |d_n|)$ and $\mathrm{sign}(D) = \mathrm{diag}(\mathrm{sign}(d_1), \ldots, \mathrm{sign}(d_n))$.

2 Passivity-preserving balanced truncation

In this section, we present a passivity-preserving balanced truncation method for circuit equations. This method is based on the fact that the transfer function $G(s)$ is positive real if and only if the Moebius-transformed function

$$\mathscr{G}(s) = \mathscr{M}(G(s)) := \big(I - G(s)\big)\big(I + G(s)\big)^{-1}$$

is *bounded real*, i.e., $\mathscr{G}$ is analytic in $\mathbb{C}_+$ and $I - \mathscr{G}(s)\mathscr{G}^T(\overline{s})$ is positive semidefinite for all $s \in \mathbb{C}_+$, see [2]. Note that for $G(s) = C(sE - A)^{-1}B + D$ with a nonsingular matrix $I + D$, the transfer function $\mathscr{G}(s) = \mathscr{M}(G(s))$ can be represented as $\mathscr{G}(s) = \mathscr{C}(s\mathscr{E} - \mathscr{A})^{-1}\mathscr{B} + \mathscr{D}$, where

$$\begin{aligned} &\mathscr{E} = E, \quad \mathscr{A} = A - B(I+D)^{-1}C, \quad \mathscr{B} = -\sqrt{2}B(I+D)^{-1}, \\ &\mathscr{C} = \sqrt{2}(I+D)^{-1}C, \qquad \mathscr{D} = (I-D)(I+D)^{-1}. \end{aligned} \tag{4}$$

For system (1), (2), a passive reduced-order model (3) can be computed by the model reduction method presented in [11, 13]. First, we consider the Moebius-transformed system $\mathscr{G} = \mathscr{M}(G)$ and apply a bounded real balanced truncation method to $\mathscr{G}$, i.e., to (4). The obtained bounded real reduced-order system $\tilde{\mathscr{G}}$ is then transformed into $\tilde{G} = \mathscr{M}(\tilde{\mathscr{G}})$ which is positive real.

2.1 Bounded real balanced truncation

The bounded realness of $\mathscr{G}$ implies that $\mathscr{G}$ is proper, i.e., there exists $M_0 = \lim_{s\to\infty} \mathscr{G}(s)$. Furthermore, for E, A, B and C as in (2), the *projected Lur'e equations* [1]

$$\begin{aligned} &E\,X\,(A-BC)^T + (A-BC)\,XE^T + 2P_l BB^T P_l^T = -2K_c K_c^T, \\ &EXC^T - P_l BM_0^T = -K_c J_c^T, \quad J_c J_c^T = I - M_0 M_0^T, \quad X = P_r X P_r^T \geq 0, \end{aligned} \tag{5}$$

and

$$\begin{aligned} &E^T Y(A-BC) + (A-BC)YE + 2P_r^T C^T C P_r = -2K_o^T K_o, \\ &-E^T YB + P_r^T C^T M_0 = -K_o^T J_o, \quad J_o^T J_o = I - M_0^T M_0 \quad Y = P_l^T Y P_l \geq 0, \end{aligned} \tag{6}$$

are solvable for $X \in \mathbb{R}^{n,n}$, $K_c \in \mathbb{R}^{n,m}$, $J_c \in \mathbb{R}^{m,m}$ and $Y \in \mathbb{R}^{n,n}$, $K_o \in \mathbb{R}^{m,n}$, $J_o \in \mathbb{R}^{m,m}$, respectively, see [13]. Here, P_r and P_l are the projectors onto the right and left deflating subspaces of the pencil $\lambda E - A + BC$ corresponding to the finite eigenvalues along the right and left deflating subspaces corresponding to the eigenvalue at infi-

[1] These equations are named after the Russian mathematician and engineer A.I. Lur'e (1901-1980). In the literature, they are also known as Kalman-Yakubovich-Popov equations [14].

nity. The minimal solutions $X_{\min}$ and $Y_{\min}$ of (5) and (6) that satisfy $0 \le X_{\min} \le X$ and $0 \le Y_{\min} \le Y$ for all symmetric solutions X and Y of (5) and (6), respectively, are called the *bounded real controllability Gramian* and the *bounded real observability Gramian* of $\mathscr{G}$.

In the bounded real balanced truncation method, we determine the Cholesky factors R and L of $X_{\min} = RR^T$ and $Y_{\min} = LL^T$, respectively, and compute the singular value decomposition

$$L^T E R = [U_1, U_2]\mathrm{diag}(\Pi_1, \Pi_2)[V_1, V_2]^T,$$

where $[U_1, U_2]$ and $[V_1, V_2]$ have orthonormal columns, $\Pi_1 = \mathrm{diag}(\pi_1 I_{l_1}, \ldots, \pi_r I_{l_r})$ and $\Pi_2 = \mathrm{diag}(\pi_{r+1} I_{l_{r+1}}, \ldots, \pi_q I_{l_q})$ with $\pi_1 > \ldots > \pi_r > \pi_{r+1} > \ldots > \pi_q$. The values π_j are called the *characteristic values* of $\mathscr{G}$. They determine the importance of state variables. A reduced-order model for $\mathscr{G} = [\mathscr{E}, \mathscr{A}, \mathscr{B}, \mathscr{C}, I]$ as in (4) can be computed by projection onto the left and right subspaces corresponding to the dominant characteristic values. Such a model is given by $\tilde{\mathscr{G}} = [\tilde{\mathscr{E}}, \tilde{\mathscr{A}}, \tilde{\mathscr{B}}, \tilde{\mathscr{C}}, I]$ with

$$\tilde{\mathscr{E}} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \qquad \tilde{\mathscr{A}} = \begin{bmatrix} W^T(A - BC)T & 0 \\ 0 & I \end{bmatrix},$$

$$\tilde{\mathscr{B}} = \begin{bmatrix} -\sqrt{2}W^T B \\ B_\infty \end{bmatrix}, \qquad \tilde{\mathscr{C}} = \begin{bmatrix} \sqrt{2}CT, & C_\infty \end{bmatrix},$$

where $W = LU_1\Pi_1^{-1/2}$, $T = RV_1\Pi_1^{-1/2}$, and the matrices B_∞ and C_∞ are chosen such that $I - M_0 = C_\infty B_\infty$.

2.2 Application to circuit equations

By exploiting the structure of circuit equations, the model reduction procedure presented above can be made more efficient and accurate. Since the MNA matrices in (2) satisfy

$$E^T = S_{\mathrm{int}} E S_{\mathrm{int}}, \qquad A^T = S_{\mathrm{int}} A S_{\mathrm{int}}, \qquad B^T = S_{\mathrm{ext}} C S_{\mathrm{int}},$$

where $S_{\mathrm{int}} = \mathrm{diag}(I_{n_\eta}, -I_{n_\lambda}, -I_{n_\nu})$ and $S_{\mathrm{ext}} = \mathrm{diag}(I_{n_l}, -I_{n_\nu})$, we find that

$$P_l = S_{\mathrm{int}} P_r^T S_{\mathrm{int}}, \qquad X_{\min} = S_{\mathrm{int}} Y_{\min} S_{\mathrm{int}} = S_{\mathrm{int}} L L^T S_{\mathrm{int}}^T = RR^T.$$

Thus, for the linear circuit equations (1), (2), it is enough to compute only one projector and solve only one projected Lur'e equation. Another projector and also the solution of the dual Lur'e equation are given for free. Furthermore, we can show that $L^T E R = L^T E S_{\mathrm{int}} L$ is symmetric. Then the characteristic values π_j can be computed from an eigenvalue decomposition of $L^T E S_{\mathrm{int}} L$ instead of a more expensive singular value decomposition. Finally, using the symmetry of $(I - M_0)S_{\mathrm{ext}}$, we can determine B_∞ and C_∞ from the eigenvalue decomposition of $(I - M_0)S_{\mathrm{ext}}$.

Summarizing, we obtain the following PAssivity-preserving Balanced Truncation method for Electrical Circuits (PABTEC).

Algorithm 1 *Passivity-preserving balanced truncation for electrical circuits.*
Given $G = [E, A, B, C]$ as in (2), compute a reduced-order model $\tilde{G} = [\tilde{E}, \tilde{A}, \tilde{B}, \tilde{C}]$.

1. Compute the Cholesky factor L of $Y_{\min} = LL^T$ that is the minimal solution of the projected Lur'e equation (6).
2. Compute the eigenvalue decomposition
$$L^T E S_{\text{int}} L = [U_1, U_2]\text{diag}(\Lambda_1, \Lambda_2)[U_1, U_2]^T,$$
where $[U_1, U_2]$ is orthogonal, $\Lambda_1 = \text{diag}(\lambda_1 I, \ldots, \lambda_r I)$, $\Lambda_2 = \text{diag}(\lambda_{r+1} I, \ldots, \lambda_q I)$ and $|\lambda_1| > \ldots > |\lambda_r| > |\lambda_{r+1}| > \ldots > |\lambda_q|$.
3. Compute the eigenvalue decomposition $(I - M_0)S_{\text{ext}} = U_0 \Lambda_0 U_0^T$, where U_0 is orthogonal and $\Lambda_0 = \text{diag}(\hat{\lambda}_1, \ldots, \hat{\lambda}_m)$.
4. Compute the reduced-order system

$$\begin{gathered}\tilde{E} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \qquad \tilde{A} = \frac{1}{2}\begin{bmatrix} 2W^T AT & \sqrt{2}W^T BC_\infty \\ -\sqrt{2}B_\infty CT & 2I - B_\infty C_\infty \end{bmatrix}, \\ \tilde{B} = \frac{\sqrt{2}}{2}\begin{bmatrix} \sqrt{2}W^T B \\ -B_\infty \end{bmatrix} \qquad \tilde{C} = \frac{\sqrt{2}}{2}\begin{bmatrix} \sqrt{2}CT, C_\infty \end{bmatrix},\end{gathered} \tag{7}$$

where $B_\infty = S_0 |\Lambda_0|^{1/2} U_0^T S_{\text{ext}}$, $C_\infty = U_0 |\Lambda_0|^{1/2}$, $S_0 = \text{sign}(\Lambda_0)$,
$W = LU_1 |\Lambda_1|^{-1/2}$, $T = S_{\text{int}} L U_1 S_1 |\Lambda_1|^{-1/2}$, $S_1 = \text{sign}(\Lambda_1)$.

One can show that the reduced-order system (7) is passive and reciprocal [13]. Furthermore, we can estimate the $\mathbb{H}_\infty$-norm of the error defined as

$$\|\tilde{G} - G\|_{\mathbb{H}_\infty} = \sup_{s \in \mathbb{C}_+} \|\tilde{G}(s) - G(s)\|,$$

where $\|\cdot\|$ denotes the spectral matrix norm. If $\|I + G\|_{\mathbb{H}_\infty}(\pi_{r+1} + \ldots + \pi_q) < 1$, then we have the following error bound

$$\|\tilde{G} - G\|_{\mathbb{H}_\infty} \leq \frac{\|I + G\|^2_{\mathbb{H}_\infty}(\pi_{r+1} + \ldots + \pi_q)}{1 - \|I + G\|_{\mathbb{H}_\infty}(\pi_{r+1} + \ldots + \pi_q)}, \tag{8}$$

see [11] for details.

3 Computation of the bounded real Gramian

If $I - M_0^T M_0$ is nonsingular, then $I - M_0 M_0^T$ is also nonsingular and the projected Lur'e equation (6) can be rewritten as the projected algebraic Riccati equation

$$E^T Y \hat{A} + \hat{A}^T Y E + E^T Y \hat{B}\hat{B}^T Y E + P_r^T \hat{C}^T \hat{C} P_r = 0, \quad Y = P_l^T Y P_l, \tag{9}$$

where $\hat{A} = A - BC - 2P_l B(I - M_0^T M_0)^{-1} M_0^T C P_r$, $\hat{B} = \sqrt{2} P_l B J_o^{-1}$, $\hat{C} = \sqrt{2} J_c^{-1} C$, $J_o^T J_o = I - M_0^T M_0$ and $J_c J_c^T = I - M_0 M_0^T$. One can show that the minimal solution $Y_{\min}$ of (6) is at least a semi-stabilizing solution of (9) in the sense that all the finite eigenvalues of $\lambda E - \hat{A} - \hat{B}\hat{B}^T Y_{\min} E$ are in the closed left half-plane. Thus, the bounded real Gramian $Y_{\min}$ can be computed by solving (9) via Newton's method.

Algorithm 2 *Newton's method for the projected Riccati equation.*
Given E, $\hat{A} \in \mathbb{R}^{n,n}$, $\hat{B} \in \mathbb{R}^{n,m}$, $\hat{C} \in \mathbb{R}^{m,n}$, projectors P_r, P_l and a stabilizing initial guess Y_0, compute an approximate solution of the projected Riccati equation (9).

FOR $j = 1, 2, \ldots, j_{\max}$

1. Compute $K_j = \hat{B}^T Y_{j-1} E$ and $A_j = \hat{A} + \hat{B} K_j$.
2. Solve the projected Lyapunov equation

$$E^T Y_j A_j + A_j^T Y_j E = -P_r^T (\hat{C}^T \hat{C} - K_j^T K_j) P_r, \qquad Y_j = P_l^T Y_j P_l.$$

END FOR

Similarly to the standard state space case [15, 16], one can show that if all the finite eigenvalues of $\lambda E - \hat{A}$ have negative real part, then starting with $Y_0 = 0$, all $\lambda E - A_j$ have finite eigenvalues in the open left half-plane only and $\lim_{j\to\infty} Y_j = Y_{\min}$. Some difficulties may occur if the pencil $\lambda E - \hat{A}$ has eigenvalues on the imaginary axis. This problem remains for future work.

If the eigenvalues of $Y_{\min}$ decay to zero very rapidly, then $Y_{\min}$ can be well approximated by a matrix of low rank. Such a low-rank approximation can be computed in factored form $Y_{\min} \approx \tilde{L}\tilde{L}^T$ with $\tilde{L} \in \mathbb{R}^{n,k}$, $k \ll n$. To determine the low-rank factor $\tilde{L}$ we can use the same approach as in [17]. Starting with $Y_{1,0} = Y_0$ and $Y_{2,0} = 0$, in each Newton iteration we compute $K_j = \hat{B}^T (Y_{1,j-1} - Y_{2,j-1}) E$, $A_j = \hat{A} + \hat{B} K_j$ and then solve two projected Lyapunov equations

$$E^T Y_{1,j} A_j + A_j^T Y_{1,j} E = -P_r^T \hat{C}^T \hat{C} P_r, \qquad Y_{1,j} = P_l^T Y_{1,j} P_l, \tag{10}$$

$$E^T Y_{2,j} A_j + A_j^T Y_{2,j} E = -P_r^T K_j^T K_j P_r, \qquad Y_{2,j} = P_l^T Y_{2,j} P_l, \tag{11}$$

for the low-rank factors $L_{1,j}$ and $L_{2,j}$ such that $Y_{1,j} \approx L_{1,j} L_{1,j}^T$ and $Y_{2,j} \approx L_{2,j} L_{2,j}^T$, respectively. Once the convergence is observed, an approximate solution $Y_{\min} \approx \tilde{L}\tilde{L}^T$ of the projected Riccati equation (9) can be computed in factored form by solving the projected Lyapunov equation

$$E^T Y \hat{A} + \hat{A}^T Y E = -P_r^T \hat{C}_0^T \hat{C}_0 P_r, \qquad Y = P_l^T Y P_l \tag{12}$$

with $\hat{C}_0 = [\hat{C}^T,\ E^T (Y_{1,j_{\max}} - Y_{2,j_{\max}}) \hat{B}\,]^T$. For computing low-rank factors of the solutions of the projected Lyapunov equations (10)–(12), we can use the generalized alternating direction implicit method [18]. Note that in this method we need to compute the products $(E^T + \tau A_j^T)^{-1} v$ with $\tau \in \mathbb{C}_-$ and $v \in \mathbb{R}^n$. Taking into account that $E + \tau A_j = E + \tau(A - BC) - \hat{B}\hat{K}_j$ with the low-rank matrices $\hat{B} \in \mathbb{R}^{n,m}$ and $\hat{K}_j = \tau(J_o^{-T} M_0^T C P_r - K_j) \in \mathbb{R}^{m,n}$ we can use the Sherman-Morrison-Woodbury formula [19, Section 2.1.3] to compute these products as

$$(E^T + \tau A_j^T)^{-1} v = v_1 + M_{\hat{K}} \left((I_m - \hat{B}^T M_{\hat{K}})^{-1} \hat{B}^T \right) v_1,$$

where $v_1 = (E^T + \tau(A - BC)^T)^{-1} v$ and $M_{\hat{K}} = (E^T + \tau(A - BC)^T)^{-1} \hat{K}_j^T$. The latter can be determined by solving linear systems with the sparse matrix $E^T + \tau(A - BC)^T$

either by computing sparse LU factorization or by using iterative Krylov subspace methods [20].

A major difficulty in the numerical solution of the projected Lyapunov and Riccati equations with large matrix coefficients is that the matrix M_0 and the spectral projectors P_l and P_r are required. Fortunately, we can exploit the structure of the MNA matrices (2) to construct the required projectors in explicit form using a matrix chain approach from [21]. Furthermore, we can obtain an explicit formula for the matrix M_0 and derive necessary and sufficient conditions for invertibility of $I - M_0^T M_0$ in terms of the circuit topology, see [13] for details.

4 Numerical example

In this section, we present some results of numerical experiments to demonstrate the feasibility of the PABTEC method.

Example This example describing a three-port RC circuit was provided by NEC Laboratories Europe. We have a passive system of order $n = 2007$. The minimal solution of the projected Riccati equation (9) was approximated by a low-rank matrix $Y_{\min} \approx \tilde{L}\tilde{L}^T$ with $\tilde{L} \in \mathbb{R}^{n,118}$ using Newton's method. Figure 1 shows that the characteristic values decay rapidly, so we can expect a good approximation by a reduced-order model. The original system was approximated by a model of order $\ell = 44$. The spectral norms of the frequency responses $\|G(\mathrm{i}\omega)\|$ and $\|\tilde{G}(\mathrm{i}\omega)\|$ for a frequency range $\omega \in [1, 10^{15}]$ are presented in Figure 2. We also display there the absolute error $\|\tilde{G}(\mathrm{i}\omega) - G(\mathrm{i}\omega)\|$ and the error bound (8).

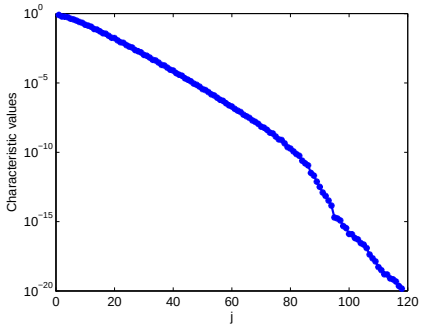

Fig. 1 RC circuit: characteristic values of $\mathscr{G}$.

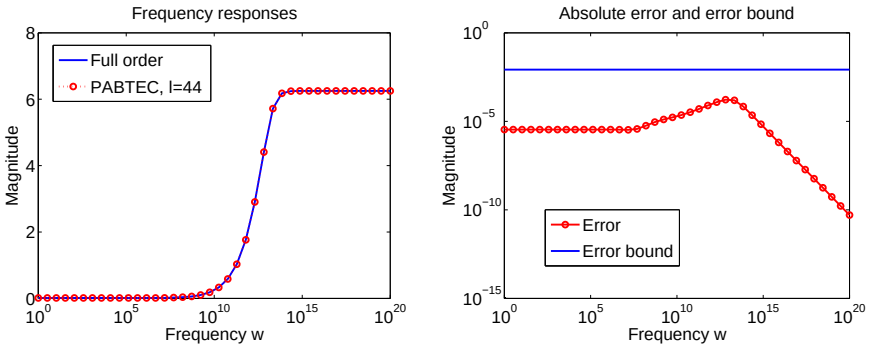

Fig. 2 RC circuit: (left) the frequency responses of the original and the reduced-order systems; (right) the absolute error and error bound.

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