1 752 730 93-02 34D08 34H05 37N35 47D06 47N70 49-02 93B29
Colonius, Fritz (D-AGSB-MU9; **Kliemann, Wolfgang** (1-IASU)
The dynamics of control
With an appendix by Lars Grüne.
Systems & Control: Foundations & Applications
Birkhäuser Boston, Inc., Boston, MA, 2000, xii+629 pp.$79.95.
ISBN 0-8176-3683-8

A continuous-time, deterministic, finite-dimensional control system is a particular type of ordinary differential equation that contains a functional parameter (called a control) which one tries to manipulate (usually within preassigned constraints) to effect some desirable system behavior (e.g., drive the system back to an equilibrium from which it may have strayed because of external disturbances, or cause the system to track a pre-assigned trajectory). Since the theory of dynamical systems has proved to be a profitable framework for the study of the qualitative behavior of ordinary differential equations, one might assume that control theorists would have long ago widely adopted a dynamical-systems approach in the study of their field (to be clear, perhaps we should state that by "dynamical-systems approach" we mean the study of control problems within the specific context of a group or semigroup acting on a set). While a few control theorists have occasionally employed or made reference to this approach, for various reasons the systematic and fundamental use of the dynamical-systems framework in control theory has been slow to materialize. These reasons stem in part from control theory's deep roots in the more applied engineering disciplines and the rapid successes of other branches of abstract mathematics (such as algebraic and differential geometry) in making fundamental contributions to our understanding of control systems. However, with the recent heightened interest in "chaotic" systems, which occur in many important applications such as turbulent fluid flow and chemical reactions, one is led naturally to questions of controlling such systems, and for these types of questions the dynamical-systems approach becomes increasingly difficult to ignore. Consequently it seems appropriate to strengthen the interactions between dynamicists and control theorists, and the book under review appears to be a timely and comprehensive first step in that direction. We say "first step" here because, in the authors' own words, "We intend this book as a report about ongoing research, not as a presentation of a complete theory. As such it leaves more questions open than it solves."Nevertheless, this book is particularly useful because it collects in one place a large number of results that are essential for an understanding of the dynamical-systems approach, but which heretofore were scattered about in the research literature. The authors have endeavored to make the book accessible to a wide audience of readers including dynamicists, control theorists, control engineers, and mathematically inclined scientists.

In somewhat more technical language, the authors focus on connections between dynamical systems, control systems, and perturbed systems in the following fashion. Consider an autonomous ordinary differential equation of the form (N) $\dot{x}(t) = X_0(x(t))$ ("N" is for "nominal" system) and consider the related (affine) control system (CAS) $\dot{x}(t) = X_0(x(t)+\sum_{i=1}^{m} u_i(t)X_i(x(t))$,Here $X_0, X_1, ..., X_m$ are smooth vector fields (defined on $\mathbb{R}^n$ or, more generally, on a differentiable manifold M) and the $u_i, i = 1, ..., m$, are real-valued functions of $t \in \mathbb{R}$; i.e., $u = (u_1, ..., u_m) \in \mathcal{U} = \{u : \mathbb{R} \to U \subset \mathbb{R}^m\}$where the set of control values $U \subset \mathbb{R}^m$ is preassigned. The authors focus on two fundamental interpretations of the affine control system (CAS) throughout their book: (i) (CAS) can be viewed as a global, time-dependent perturbation of the nominal system (N). Then for a given control function $u \in \mathcal{U}$ one can study the limit behavior of the trajectories of (CAS) as $t \to \infty$ in comparison to the corresponding behavior of the trajectories of (N). If the control set U is appropriately small, then this approach fits into the standard framework of perturbation theory. (ii) (CAS) can be viewed as the fundamental object of interest. The controls are not fixed, but rather are allowed to vary (say within the family of locally integrable, U-valued functions), and one seeks to find controls that will generate trajectories with certain desired properties. Of course, the first approach is the traditional one for dynamicists while the second is the traditional one for control theorists. To view a control system as a dynamical object, the authors proceed as follows. For $x \in M$ and $u \in \mathcal{U}$ the unique solution of (CAS) that passes through x at time $t = 0$ is denoted by $t \mapsto \varphi(t, x, u)$. The authors assume completeness of the system throughout (i. e., $\varphi(t, x, u)$ is defined for all $t \in \mathbb{R}$). Such completeness would hold, for example, if the state space M were a compact manifold or if appropriate growth conditions were imposed on the system vector fields. In addition, the authors assume that the control value set $U \subset \mathbb{R}^m$ is compact and convex. Then the space of controls U is in fact a subset of $L_\infty(\mathbb{R}, \mathbb{R}^m)$, the set of bounded and measurable (as opposed to merely locally integrable) functions from $\mathbb{R}$ into $\mathbb{R}^m$. Because of the duality $L_\infty(\mathbb{R}, \mathbb{R}^m) = (L_1(\mathbb{R}, \mathbb{R}^m))^*$ we can endow $\mathcal{U}$ with the weak* topology, in which case standard results from functional analysis imply that $\mathcal{U}$ is convex and (weak*) compact. One then defines a shift mapping $\Theta : \mathbb{R} \times \mathcal{U} \to \mathcal{U}$ on $\mathcal{U}$ by $\Theta(t,u)(\tau) = \Theta_t(u)(\tau) = u(t+\tau)$ and the corresponding control flow $\Phi : \mathbb{R} \times \mathcal{U} \times M \to \mathcal{U} \times M$ by $\Phi(t,u,x) = (\Theta(t,u), \varphi(t,x,u))$. $\varphi\varphi$ It can be seen without much difficulty that $\Phi_{t+s} = \Phi_t \circ \Phi_s$ and that Φ is continuous in the indicated topologies (though it should be noted that continuity with respect to the control in the weak* topology requires both the completeness assumption and the affine structure of the control system). In this fashion the control system (CAS) induces a continuous dynamical system on $\mathcal{U} \times M$.

Having established the framework in which the authors carry out their exposition, we next outline the contents of the book. The authors begin with a preliminary chapter on "Dynamics, perturbations, and control", which sets the tone of the development and provides an overview of the topics to be discussed in the sequel. The specific topics discussed are: perturbations of complex behavior, approximation of complex systems, generic behavior of perturbations, stability boundaries and multistability, reachability in control systems, linear and nonlinear stability radii, stabilization of bilinear systems, and the Lyapunov spectrum of matrices. These topics are interesting because they are firmly rooted in either control theory or dynamical-systems theory, and because for the most part they include important open problems for which the authors provide partial solutions later in the text (partial solutions are nice because they whet the appetite, but still leave interesting work for the rest of us).The remainder of the book is broken down into three major parts: (i) global theory (including the notions of control sets and chain control sets, recurrence and ergodic theory, chain controllability, genericity, and perturbations); (ii) linearization theory (including the Morse and Lyapunov spectra, bilinear systems on vector bundles, and linearization at a singular point); (iii) applications (including one-dimensional systems, examples of global behavior from chemical engineering and biology, examples of the spectrum, stability radii and robust stability, open and closed loop stabilization, and the dynamics of perturbations).Finally, there are four extensive appendices dealing with geometric control theory, dynamical systems, the numerical computation of orbits, and the numerical computation of the spectrum (the latter was contributed by Lars Grüne). These appendices should help to fill gaps in the varying backgrounds of many potential readers and thereby make the book accessible to a wide audience Each chapter concludes with a section of explanatory notes and references that point the reader to related topics in the research literature. Also included is a comprehensive bibliography with more than 300 entries.

The authors have obviously invested a considerable amount of effort in the preparation of this book. The topics are well organized, the exposition is clear, and there are ample examples to illustrate the theoretical results. This book is highly recommended for both researchers and graduate students in dynamical systems, control theory, and related topics.

Reviewed by Kevin A. Grasse

Mathematical Reviews 2001