The Dynamics of Control

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Control theory and the theory of dynamical systems are traditional fields of mathematical research which have developed into powerful theories with important applications in science and technology. While the two theories are obviously linked in various ways, their development has largely been independent of each other and research incorporating both concepts from control and dynamical systems theory have been relatively sparse. In this context, the authors of the book under review have done some pioneering work over a period of about ten years in bringing together control-theoretic notions and basic concepts from the theory of dynamical systems. As an appropriate link between control and dynamical systems they have introduced the so-called control flow, an infinite dimensional dynamical system acting on the product of the space of admissible controls and the state space. Using this control-dynamic device, they have studied control-theoretic topics such as controllability regions and their domains of attraction, robust stability and stability radii, and open loop as well as feedback stabilization for nonlinear systems. Applications to dynamical systems include time-varying perturbations of nominal systems, new spectral concepts, and persistence and continuity results for attractors and their spectra.

In this book Colonius and Kliemann present an up-to-date account of their work on the dynamical theory of control. The systems they consider are ordinary differential equations $\dot{x} = X_0(x)$ and their control-theoretic counterparts $\dot{x} = X_0(x) + \sum_{i=1}^{m} u_i(t)X_i(x)$ where the $X_0, X_1, \ldots, X_m$ are C^∞-vector fields on a connected Riemannian C^∞-manifold M and the $u_1, \ldots, u_m$ are real-valued functions of $t \in \mathbb{R}$ with $u = (u_1, \ldots, u_m) \in \mathcal{U} = \{u : \mathbb{R} \to U\}$ where U is a given subset of $\mathbb{R}^m$. They suppose throughout that for each $u \in \mathcal{U}$ and $x \in M$ the solution $\varphi(t, x, u)$ of the control system starting in x at $t = 0$ is unique and exists for all $t \in \mathbb{R}$. Of the different ways to interpret the role of the functions $u \in \mathcal{U}$ relative to the nominal system $\dot{x} = X_0(x)$ in this book both a perturbation and a control-theoretic point of view is taken, i.e., the functions $u \in \mathcal{U}$ are interpreted as (deterministic) perturbations or as controls. In the former case the focus is on the limit behavior of the trajectories of the perturbed system relative to the nominal system, and in the latter case the problem is to choose from among the admissible controls a particular function in such a way that the system exhibits a certain prescribed behavior.

It is a feature of this book that control-theoretic and perturbation problems are treated in a unifying way by associating to a control/perturbation system a dynamical system over the space of control/perturbation functions. In this way, a variety of techniques from the theory of dynamical systems (e.g. mixing, chain recurrence, Morse decompositions, invariant measures, Lyapunov exponents, invariant manifolds) as well as ideas from control theory (e.g. accessibility, reachability, control sets, optimal control) can be used to analyze the associated dynamical systems. This methodology allows new results in control and perturbation theory, some of which apply to classical problems in these areas and some to questions of current interest.

After this general description of the authors' view on the connection between dynamical and control systems we now outline the contents of this book. Following a short overview and some hints on how to use this book, the preliminary Chapter 2 presents eight classical and current problems which serve as a motivation for the theory and provide a body of examples that will be discussed in later chapters. These topics concern 1. Perturbations of Complex Behavior, 2. Approximation of Complex Systems, 3. Generic Behavior of Perturbations, 4. Stability Boundaries and Multistability, 5. Reachability in Control Systems, 6. Linear and Nonlinear Stability Radii, 7. Stabilization of Bilinear Systems and 8. The Lyapunov Spectrum of Matrices.

The main body of the book consists of three parts: I. Global Theory, II. Linearization Theory and III. Applications.

In Part I (Chapters 3 and 4) the global behavior of nonlinear systems is investigated in term of control sets and control flows. The notion of control set is introduced in Chapter 3 as a maximal subset of the state space where complete controllability holds. Then the basic properties of control sets are derived and illustrated by means of a number of examples, and the reachability order on control sets is described. Furthermore, two variants of control sets are discussed, control sets that are maximal within a subset of the state space, and chain control sets which are based on a weaker notion of controllability allowing for arbitrarily small jumps.

The key notion of this book, the control flow $\Phi : \mathbb{R} \times \mathcal{U} \times M \to \mathcal{U} \times M$, is introduced in Chapter 4 as $\Phi(t, u, x) := (\Theta(t, u), \varphi(t, x, u))$ where $\Theta(t, u)$ is the shift on $\mathcal{U}$ defined by $\Theta(t, u)(s) = u(t + s)$ and $\varphi(t, x, u)$ the solution of the control system with $\varphi(0, x, u) = x$. In order for $\mathcal{U}$ to become a topological space it is vital that the control functions appear affinely in the given system and that the control range $U \subset \mathbb{R}^m$ is compact and convex. Under these assumptions $\mathcal{U}$ is a subset of $L_\infty(\mathbb{R}, \mathbb{R}^m)$, the space of bounded and measurable functions from $\mathbb{R}$ to $\mathbb{R}^m$, and because of the duality relation $L_\infty(\mathbb{R}, \mathbb{R}^m) = (L_1(\mathbb{R}, \mathbb{R}^m))^*$ the set $\mathcal{U}$ may be endowed with the weak*-topology. It should be noted that in this construction not only piecewise constant controls are taken into account but measurable ones, because this gives $\mathcal{U}$ the structure of a compact metric space, a property which is convenient from a dynamical systems point of view. In fact, it is shown that the shift Θ on $\mathcal{U}$ is continuous, topologically mixing, and transitive and that the periodic functions are dense in $\mathcal{U}$. Thus, the shift also has sensitive dependence on initial conditions and therefore it is chaotic in the sense of Devaney. In addition it is chain recurrent. Further topics covered in Chapter 4 concern the relation between control sets and the topologically mixing components of the control flow, ergodic properties of the control flow, chain controllability and inner pairs, the generic behavior of the control flow as $t \to \pm\infty$ and the behavior of control sets under perturbations of the control range.

Part II (Chapters 5-7) contains the basic results on linearized systems. It begins with general linear flows on vector bundles and their spectral theory, including the Morse spectrum and other spectral concepts such as the Lyapunov, the Sacker-Sell and the Oseledets spectrum. It also contains an abstract invariant manifold theorem. Then these results are specialized to bilinear and linearized control systems on vector bundles, including a characterization of the Lyapunov spectrum in terms of the Floquet and the Morse spectrum and some results on local stable manifolds. Finally, the most specialized case, the linearization at a singular point, is singled out in a separate chapter because this is all what is needed for (robust) stability and stabilization of linear and (classical) bilinear control systems in later chapters.

Part III (Chapters 8-13) deals with examples of local and global analysis of systems. First one-dimensional systems are considered for which explicit constructions of (chain) control sets and spectra are possible, and then the global behavior of nonlinear systems is studied with respect to controllability and reachability properties. In Chapter 10 it is shown how the Lyapunov spectrum of control systems can be computed numerically. The examples include several two- and three-dimensional parameter-controlled linear oscillators and a continuous flow stirred tank reactor. The results and algorithms described here are used in the remaining chapters to illustrate robustness and stabilization properties of control systems and to explain the use of control-theoretic concepts in the study of dynamical systems. In Chapter 11 the focus is on linear systems with time-varying perturbations from the point of view of robust stability and stabilizability, and Chapter 12 deals with open and closed loop stabilization. Finally, in Chapter 13 some consequences of the theory developed in this book are drawn for persistence and continuity of attractors and their spectra, including applications to the Lorenz system and a model for ship roll motion.

The final Part IV of this book consists of four extensive appendices: A. Geometric Control Theory, B. Dynamical Systems, C. Numerical Computation of Orbits and D. Computation of the Spectrum (by Lars Grüne).

In summary, the book under review represents a systematic and comprehensive description of the dynamical theory of control as the authors have developed it over more than a decade. It gives an up-to-date account of their research combining ideas and concepts from control theory and the theory of dynamical systems. As this approach of linking two different but related fields turns out to be very fruitful and beneficial for both areas, this book is a valuable addition to the mathematical literature. Moreover, it provides a significant amount of material for further research in the fields of control and dynamical systems, as demonstrated e.g. by means of eight open questions of current interest in the Introduction of this book. For readers with different backgrounds and levels of experience the authors provide various support to make this mathematically ambitious book accessible. At the end of each chapter they present Notes that refer to related topics in the research literature, the four Appendices exhibit much of the background material that might not be available to all of the potential readers, and the extensive Bibliography contains 333 references. All in all, this book is highly recommend reading for researchers and graduate students in the fields of control theory and dynamical systems, but also for control engineers and mathematically inclined scientists.

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