Zentralblatt MATH Database 1931 – 2015

Zbl 1306.37001

Colonius, Fritz; Kliemann, Wolfgang

Dynamical systems and linear algebra. (English)

Graduate Studies in Mathematics 158. Providence, RI: American Mathematical Society (AMS). xv, 284 p. \$ 67.00 (2014). ISBN 978-0-8218-8319-8/hbk

This book is to devoted to dynamical systems whose behavior is determined by real quadratic matrices $A \in \mathbb{R}^{d\times d}$. The corresponding systems are described by linear differential equations of types $\dot{x} = Ax$, $A \in \mathbb{R}^{d\times d}$, or $\dot{x} = A(t)x$ $A(t) \in \mathbb{R}^{d\times d}$, $t \in \mathbb{R}$ in the case of continuous time t and with linear difference equation $x_{n+1} = Ax_n$, $A \in \mathbb{R}^{d\times d}$ or $x_{n+1} = A_n x_n$, $A_n \in \mathbb{R}^{d\times d}$, $n \in \mathbb{Z}$ in the case of discrete time n. It is natural that linear algebra plays a key role in the analysis of such systems; moreover this analysis leads to the study, characterization and modification of many objects in linear algebra, such as similarity matrices, spectral analysis of matrices and their special classes, and so on. As a result in many linear algebra textbooks (e.g., by F. R. Gantmacher or R. A. Horn and C. R. Johnson) one can find basic results about linear differential and difference equations, however, the serious and fundamental account of the theory of these equations from the view-point of dynamical systems does not exist. And this book is among the first ones dealing with the curious interplay between linear algebra and the theory of dynamical systems. Of course, it is impossible to cover the immensity of this play, but this book is a nice step in the corresponding direction. The book contains 11 chapters divided into two parts; each chapter begins from a small introduction, which describes what one can find in the chapter, further, it runs some sections with basic text, section with exercises, and a section "Orientation, notes and references" (explanation of stated results and references). The book is completed with a bibliography (144 items) and an index.

The first part (5 chapters) of the book, "Matrices and linear dynamical systems", deals with the autonomous case. Chapter 1 "Autonomous linear differential and difference systems" presents basic results about the representation of solutions to the linear differential equation $\dot{x} = Ax$ and the decomposition of $\mathbb{R}^d$ into the direct sum of Lyapunov (and A invariant) spaces L_j; every such space consists of solutions with Lyapunov exponent λ_j, $\lambda_\ell < \ldots < \lambda_2 < \lambda_1$. This decomposition is obtained on the base of the Jordan canonical form of the matrix A. Similar results are obtained for linear difference equation $x_{n+1} = Ax_n$. Chapter 2 "Linear dynamical systems in $\mathbb{R}^d$" deals with the classification of dynamical systems generated by the equations $\dot{x} = Ax$ and $x_{n+1} = Ax_n$, $A \in \mathbb{R}^{d\times d}$ and the corresponding classification of matrices. Chapter 3 "Chain transitivity for dynamical systems" sets forth fundamental notions of the theory of dynamical systems: α- and ω-limit sets, the chain transitivity, the chain recurrent sets and describes the different relations between them, and, in particular, the preservation of chain transitive sets under conjugacies of dynamical systems. Chapter 4 "Linear systems in projective spaces" deals with dynamical systems induced on the projective space $\mathbb{P}^{d-1}$ by equations $\dot{x} = Ax$ and $x_{n+1} = A_n x_n$ with $A \in \mathbb{R}^{d\times d}$. In particular, it is established that the Lyapunov spaces of the equations $\dot{x} = Ax$ and $x_{n+1} = A_n x_n$ coincide with the preimages (under the canonical mapping $\mathbb{R}^d \to \mathbb{P}^{d-1}$) of chain components of the dynamical systems in $\mathbb{P}^{d-1}$. Chapter 5 "Linear systems on Grassmannians" is devoted to the analogous results for dynamical systems induced by equations $\dot{x} = Ax$ and $x_{n+1} = A_n x_n$ in the Grassmann manifolds of k-dimensional subspaces in $\mathbb{R}^d$ (here, only the case of continuous time is considered).

The remaining 6 chapters form the second part of the book, "Time varying matrices and linear skew product systems". Chapter 6 "Lyapunov exponents and linear skew product systems" deals with equations $\dot{x} = A(t)x$ and $x_{n+1} = A_n x_n$. The authors prove the existence of solutions for such equations, they state the existence of principal fundamental solutions $X(t, t_0)$ and $X(n, n_0)$, further, they present the definition of Lyapunov exponents for solutions and a series of properties. Further, they introduce the notion of linear skew product flow as a dynamical system $\Phi = (\theta, \varphi)$ with the state space $X = B \times \mathbb{R}^d$ such that $\mathbb{R} \times B \times \mathbb{R}^d \to B \times \mathbb{R}^d$ where $\theta: \mathbb{R} \times B \to B$ and $\varphi\colon \mathbb{R} \times B \times \mathbb{R}^d \to \mathbb{R}^d$ is linear with respect to the $\mathbb{R}^d$-argument. In the chapter, some examples of such systems are presented and among them the dynamical systems generated by equations $\dot{x} = A(t)x$ and $x_{n+1} = Ax_n$. Chapter 7 "Periodic linear differential and difference equations" deals with the special case when $A(t)$, $t \in \mathbb{R}$, or $\{A_n\}$, $n \in \mathbb{Z}$, is periodic. As is well known (Floquet theory), the equations $\dot{x} = A(t)x$ and $x_{n+1} = Ax_n$ in this case are reducible to the similar equation $\dot{x} = Bx$ and $x_{n+1} = Bx_n$ by means of a Lyapunov transformation preserving the Lyapunov exponents. The chapter offers a complete account of Floquet theory for continuous and discrete times (the continuous case is illustrated in details with the analysis of the Mathieu equation with periodic coefficients). Chapter 8 "Morse decompositions of dynamical systems" is devoted to the general theory of continuous flows on compact metric spaces within a topological framework. More exactly, the authors introduce and study the Morse decomposition for continuous flows and the relations between the notions of Morse sets, attractors, repellers, and chain transitivity. In particular, conditions for the existence of a finest Morse decomposition are presented. Chapter 9 "Topological linear systems" deals with the Morse decomposition and the Morse spectrum for linear systems $\dot{x} = A(t)x$ with the matrix-valued-function of type $A(t)x = A_0 x + \sum_{i=1}^{m} u_i(t) A_i$ (robust linear systems). The main part of the chapter is devoted to detailing the results of the previous chapter for linear skew product flows that are considered as topological flows on the vector bundles $B \times \mathbb{R}^d$ and $B \times \mathbb{P}^{d-1}$. In particular, one obtains here the full analogue (Selgrade theorem) of the theorem about coincidence of Lyapunov subspaces and preimages of the corresponding projective chain components from Chapter 4. All the constructions of the chapter are restricted to systems with continuous time. Chapter 10 "Tools from ergodic theory" deals with invariant measures of dynamical systems generated by linear maps on probability spaces and presents two classical results: Birkhoff ergodic theorem and Kingman subadditive ergodic theorem. The last Chapter 11 "Random linear dynamical systems" is devoted to systems of type $\dot{x}(t) = A(\theta_t \omega)x(t)$, $t \in \mathbb{R}, \omega \in \Omega$. Here θ is a metric dynamical system on a probability space $(\Omega, \mathcal{F}, P)$. The Oseledets Multiplicative Ergodic Theorem and Furstenberg-Kesten Theorem are basic results in this chapter. As an example, the authors consider a linear oscillator $\ddot{x}(t) + 2b\dot{x}(t) + (1 + \rho f(\theta_t \omega))x(t) = 0$ with random restorting force.

In general, the book is interesting. Its main part is written in a simple and clear manner. It contains a systematic account of the fundamental and important for applications branch of mathematics, lying on the intersection of linear algebra, the theory of differential equations, and the theory of dynamical systems. However, one can make a few remarks about the lacks of the book. For example, the authors consider two equivalent metric d and $\bar{d}$ on the projective space $\mathbb{P}^{d-1}$; but these metrics are proportional in an obvious way. Further, many constructions in this book in the nonautonomous case could be enlarged to reducible systems, systems with commuting matrices, systems with almost periodic matrices, and so on. Also absent is the phenomenon of dichotomy for different classes of equations with variable matrices. The authors themselves write that the analysis of nonlinear systems via linearization (quasi-linear systems) is omitted; the rich interplay with theory Lie groups and semigroup is not presented in this book. Nonetheless, it is undoubtedly, the book will be useful for specialists and postgraduate students.

Peter P. Zabreĭko (Minsk)

Keywords : dynamical systems; linear differential and difference equations; Lyapunov exponents, projective spaces, Grassmann linear manifolds; skew product systems; Floquet theory; α-limit sets; ω-limit sets; chain transitivity; ergodic theory; random dynamical systems

Classification :

$*$37-01 Instructional exposition (Dynamical systems and ergodic theory)

34-01 Textbooks (ordinary differential equations)

93C05 Linear control systems

15A99 Miscellaneous topics in linear algebra

34A30 Linear ODE and systems

39A06